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Modal Analysis of rotating machinery system

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ABSTRACT : *In this present work a direct-coupled rotor system was designed to analyze the dynamic behavior of rotating systems in regard to vibration parameters. The vibration parameters are amplitude, velocity, and acceleration in the vertical direction. To analyze the rotor-shaft system three methods are used. First method is a Classical method i.e. Linear Differential Method. A Non-linear mathematical equation is given for rotating system. A Linear Differential Method has solved this equation. The equation is used to calculating the critical speeds of the machines without the effects of damping. The inclusion of damping adds a huge degree of complexity in formulating and solving the problem. The main purpose of equation is to determine the actual critical speed and the corresponding amplitude, as the rotor increases from zero speed to its running speed. The amplitude values have been checked in mat-lab also. Second method is used as FEA. Third method is an Artificial Neural Network (ANN). Experimental data has taken from the literature can be trained to the network. The network computes any deviation in the displacements of rotor-shaft system. Online monitoring of the displacements can be done with the network. Further the deviation is checked for the acceptable range. If the deviation is out of range then a necessary action is suggested. A simple back propagation network is used to carry out the process. The results showed that the network could be used an analyzer of such systems. The three method results are compared. A frequency formula for simple supported beam with centrally applied load has taken. It is used to calculate the natural frequency of the system. A Modal Analysis is done in FEA to determine the Natural frequency of the system. Comparison of theoretical and FEA results compared.*

Keywords: *Critical speed,Natural frequency,Linear differential method,ArtificialNeural networks,Finite Element Analysis*

I. INTRODUCTION AND LITERATURE REVIEW

Condition-monitoring techniques for the diagnosis of faults in rotating machinery need to be improved in order to be able to identify, as quickly as possible, the many different kinds of faults that can occur in a rotor-dynamic system. Vibration-response measurements yield a great deal of information concerning any faults in a rotating machine. The identification of common mass unbalance by vibration analysis is very well developed and can be performed in many ways; however, the identification of faults such as bowed or cracked shafts, rubbing, and bearing misalignment remains relatively basic. Much research must be applied to these areas so as to devise comprehensive fault diagnosis schemes that can automatically detect any faults that may arise in a system and provide information concerning the best correction procedure to be used.

Recently, rotating machinery has been studied in greater detail. A thorough understanding of the principles of rotor dynamics is essential for engineers and scientists involved in the transportation and power-generation industries, as well as in many other fields on which we find ourselves relying to an increasing extent.

Because the analysis and design of rotating machinery are extremely critical in terms of the cost of both production and maintenance, it is not surprising that fault diagnosis of rotating machinery is a crucial aspect of the subject, one that is receiving ever more attention. As the design of rotating machinery becomes increasingly complex as a result of the rapid progress being made in technology, so must condition-monitoring strategies become more advanced in order to cope with the physical burdens being placed on the individual components of a machine. Modern condition-monitoring techniques encompass many different themes, one of the most important and informative being vibration analysis, a field in which much research has been carried out and a corresponding amount of literature produced. Using vibration analysis, the state of a machine can be constantly monitored, and detailed analyses may be made concerning the health of the machine and any faults that may be arising or may have already arisen, serious or otherwise. Common rotor-dynamic faults include self-excited vibration due to system instability and, more commonly, vibration due to some externally applied load, such as cracked or bent shafts or mass unbalance [1].

A complete rotor-bearing system can be modeled, for the purpose of dynamic analysis, by assembling the dynamic properties of its subsystems, such as the rotating components, the journal bearings the foundations, etc. These mathematical models are, in general, represented by a set of differential equations, the equations of motion. Because there are numerous approaches solving the equations of motion and numerous methods, which can be utilized for predicting the subsystem dynamic properties, the mathematical model for a rotor-bearing system can be constructed in many different forms. Lund [2] used the modal representation of shafts in modeling the rotating components of a rotor-bearing system. The rotor was supported at its static equilibrium state in the bearings, which were represented by a set of stiffness and damping coefficients. Thus the entire rotor-bearing system was treated as a non-conservative system with unsymmetrical stiffness and damping matrices. Earlier, Morton [3] used similar approaches for rotor modeling and bearing modeling. The dynamic properties of the rotor were obtained by considering the kinetic and strain energies of the rotor and its supporting structure when perturbed from the equilibrium state by a very small increment. The equations of motion were expressed in terms of the characteristic modes of the rotor as a continuous beam on rigid supports together with its free modes.

II.THEORITICAL BACKGROUND:

Introduction

A body said to be vibrate if it has to and fro motion. These are mainly due to elastic nature of the body. Whenever an elastic body such as springs, beam and a shaft are displaced from the equilibrium position by the application of external force, and released, they execute a vibratory motion. This is due to main reason that, when the body is displaced form its equilibrium position it stores the energy in the form of elastic energy or strain energy. At release of the body from loading these forces bring the body to its original position. When the body reaches to equilibrium position, the whole of the elastic energy or the strain energy is converted into kinetic energy due to which the body continues to move in the opposite direction the whole of the kinetic energy is again converted into strain energy due to which it again reaches to the equilibrium positions. In this way the vibratory motion is repeated indefinitely.

Vibration can generally be divided into two categories linear and torsional.

Linear vibration is the type most of people are familiar with, this is the shaking and movement of equipment can easily feel and hear. As a rule, linear vibrations are related to some type of mechanical problem such as imbalance or misalignment, or, advanced wear of gears or bearings. Linear vibrations tend to get worse as speeds increases, the vibration increases in both frequency and amplitude in other words the faster the tire turns the worse the imbalance becomes.

Vibration of turbomachinery can very seriously affect the functionality and profitability of industrial plants. The reduced output or unplanned shutdowns of machines are a common result of high levels of vibrations. In very simple terms, the turbomachinery consists of a rotor (with impellers/bladed disks, etc) supported on bearings and rotating in the bearing clearance space. Basically there are three forms of vibrations associated with the motion of the rotor: torsional, axial and lateral. Torsional vibration is the dynamics of the shaft in the angular/rotational direction. Normally, this is little influenced by the bearings that support the rotor. Axial vibration is the dynamics of the rotor in the axial direction and is generally not a major problem. Lateral vibration, the primary concern, is the vibration of the rotor in the lateral directions. The bearings play a huge part in determining the lateral vibrations of the rotor. In this short course, we will study the basic concepts of the lateral rotor dynamics of turbomachinery [30, 31].

The turbomachinery consists of a rotor (with impellers/bladed disks, etc) supported on bearings and rotating in the bearing clearance space. To understand the basic principles of the dynamic behavior of the rotor, let us look at a simple rotor-bearing system and then extend these principles to the more complicated real-world turbomachinery.

• ***Natural frequency*:** the frequency of vibration of a system (e.g.. rotor-bearing system) under free conditions (i.e.., without external forces). This is a function of the system. Each system has its own natural frequencies. Consider a very simple system – a mass supported by a spring, the natural frequency of this system is given by:

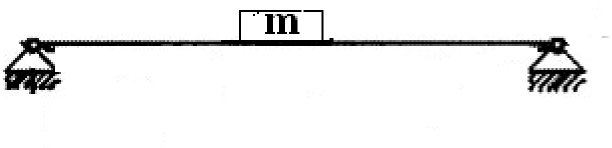
$$\omega_n = \sqrt{(k / m)}$$

Factors affecting natural frequencies:

The major factors affecting the natural frequencies of an object are the objects inertia and rigidity,Resonance,Critical speed.

Rotor supporting system:

Figure shows a classical Jeffcott rotor – a rotor with a concentrated mass at the center and supported by bearings at each end



Let us assume that the mass is concentrated at the midspan. The bearings are assumed to be rigid supports. Thus the rotor can be assumed to be simply supported. Using the theory of beams, the stiffness of the simply supported beam can be written as,

$$k = [48EI] / L^3$$
$$= [48E(\pi d^4)] / (64\ L^3)$$

Using the above equation for natural frequency, we obtain,

$$\omega_n = \sqrt{(k / m)}$$
$$= \sqrt{\{48E(\pi d^4)] / (64L^3 m)\}}$$

If we assume distributed mass of the shaft of diameter “d” and length “L”, the above equation can be reduced to,

$$\omega_n = \sqrt{(k / m)}$$
$$= \sqrt{\{[48E(\pi d^4)] / (64\ L^3) x4 / (\rho\ \pi d^2\ L)\}}$$
$$= f\ (d / L^2)$$

THEORY OF THE ROTATING SYSTEM

Let us consider the rotor shown in Figure 1 which by assumption rotates with a constant angular speed Ω.

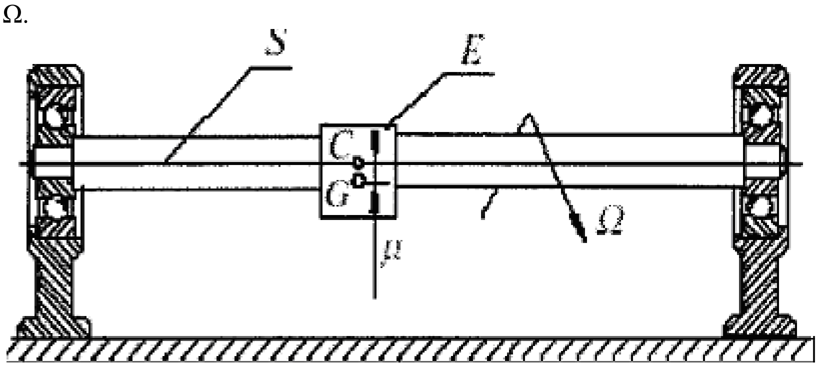


FIGURE 1 Description of the system.

The shaft *S* of the rotor is supported rigidly at its ends. Assume that the shaft can be considered massless and flexible, whereas the element *E* can be approximated by a particle of mass *m*. This particle is attached to the shaft at the center of gravity *G* of the element *E*. The center of gravity *G* is displaced by μfrom the geometrical center of the shaft cross-section *C*. The distanceμ represents the imbalance of the element *E* and can be considered to be of small magnitude. To analyze motion of this system, let us introduce the inertial system of coordinates *X*, *Y*, and *Z* as it is shown in Figure 2. The instantaneous position of the center *C* is determined by the position vector r_C . The center of gravity *G* rotates with respect to this center with the angular velocity Ω. Since the angular velocity is constant, the relative instantaneous position of the center of gravity *G* is determined by the angle Ω*t* and the imbalance μ (vector r_{GC}). The absolute position of the center of gravity *G* in Figure 2 is denoted by r_G. The vector *Fs* represents the static resultant force acting on the element *E*. *R* stands for the interaction force between the element considered and the shaft.

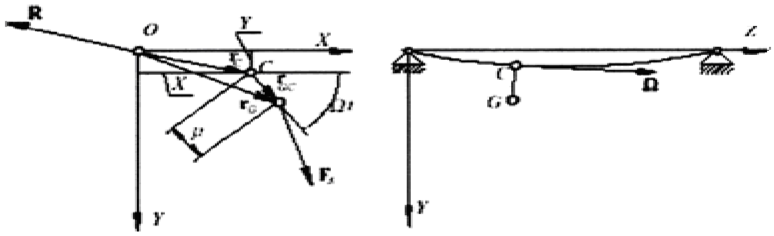


FIGURE 2 Representation of the absolute position of the center of gravity *G*.

MATHEMATICAL MODEL OF THE SYSTEM

The motion of the center of gravity *G* is governed by the Newton law

$$m\ddot{r}_G = R + F_s \qquad (1)$$

Where, according to Figure 2,

r_G = I(X+μcos (Ωt) +J(Y+ μsin (Ωt)

R = -IkX-JkY (2)

$F_S = IF_X + JF_Y$

In the above formula, *k* stands for the stiffness of the shaft at the point *C*, and *X* and *Y* are its coordinates. The introduction of Equation (2) into Equation (1) results in the following set of differential equations:

$$m(\ddot{Y} - \mu\Omega^2 \sin\Omega t) = -kY + F_Y \qquad (3)$$

or, after reorganization,

$$m\ddot{X} + kX = F_X + m\mu\Omega^2 \cos\Omega t \qquad (4)$$

$$m\ddot{Y} + kY = F_Y + m\mu\Omega^2 \sin\Omega t$$

The particular solution of the nonhomogeneous Equation (5),

$$m\ddot{X} + kX = F_X \qquad (5)$$

$$m\ddot{Y} + kY = F_Y$$

Yields the equilibrium position (*Xs;Ys):* Upon assuming the particular position in the form

$$X = X_s \qquad (6)$$

$$Y = Y_s$$

One may obtain the following formulas for the coordinates of the equilibrium position, which are usually referred to as the static deflection of the shaft.

$$X_s = \frac{F_X}{k} \qquad (7)$$

$$Y_s = \frac{F_Y}{k}$$

The total deflection of the shaft *X*, *Y* is the sum of the static deflection *Xs,Ys*, and the dynamic deflection *x*, *y* (Figure 3):

$$X = X_S + x \qquad (8)$$

$$Y = Y_S + y$$

The introduction of Equation (8) into mathematical model in Equation (4) that govern the dynamic deflections *x, y*

$$m\ddot{y} + ky = m\mu\Omega^2 \sin\Omega t \qquad (9)$$

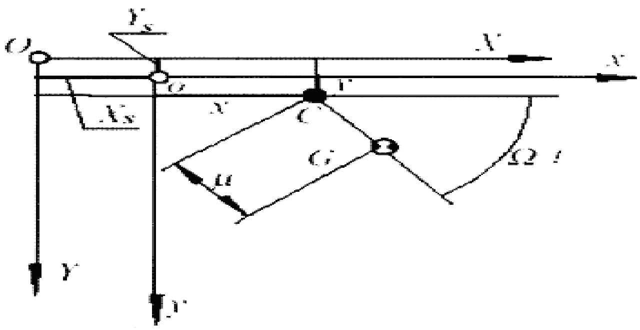


FIGURE 3 Total deflection of shaft on the planar surface.

$$\ddot{x} + \omega^2 x = q\cos\Omega t \qquad (10)$$

$$\ddot{y} + \omega^2 y = q\sin\Omega t \qquad (11)$$

$$\omega = \sqrt{\frac{k}{m}} \;, \qquad q = \mu\Omega^2 \qquad (12)$$

Upon multiplying Equation (11) by the imaginary unit *i* and adding Equations (10) and (11), one may obtain the equations of motion of the rotor in the following form:

$$\ddot{z} + \omega^2 z = q\,e^{i\Omega t} \qquad (13)$$

$$z = x + iy \qquad (14)$$

The above equation governs motion of the rotor in the stationary system of coordinates *x*, *y*, and *z*. Let us introduce the rotating system of coordinates x_R, y_R, and $z_R$as shown in Figure 4. Axis $Z_R$coincides axis *z*, and axes $x_R$and y_R, *rotate* with the constant angular velocity Ω. In terms of the complex notations, the position of the point *C* in the stationary system of coordinates *x*, *y*, and *z* is

$$z = z\,e^{i\phi} \qquad (15)$$

and in the rotating system of coordinates, it is

$$z_R = z_R e^{i(\phi-\Omega t)} = z_R e^{i\phi} e^{\Omega t} \qquad (16)$$

The introduction of Equation (15) into Equation (16) yields the relationship between coordinates of the same point in the stationary (*x*, *iy*) and the rotating (x_R, y_R,) systems of coordinates:

$$z_R = z\,e^{-i\Omega t} \qquad (17)$$

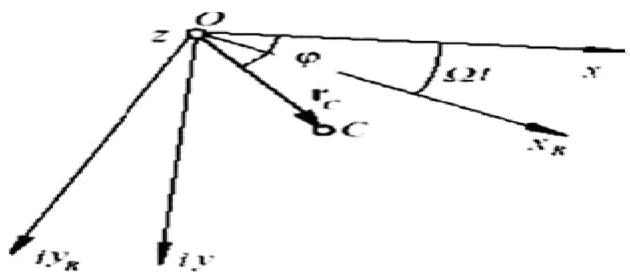


FIGURE 4 the rotating system coordinates.

The inverse transformation is

$$z = z_R e^{i\Omega t} \qquad (18)$$

Differentiating Equation (18) with respect to time, one can obtain

$$\dot{z} = \dot{z}_R e^{i\Omega t} + z_R i\Omega e^{i\Omega t} \qquad (19)$$

$$\ddot{z} = \ddot{z}_R e^{i\Omega t} + 2\dot{z}_R i\Omega e^{i\Omega t} - z_R \Omega^2 e^{i\Omega t}$$

The introduction of Equation (19) into mathematical model produces the equation of motion of the rotor in terms of the rotating system of coordinates:

$$\ddot{z}_R + 2\dot{z}_R i\Omega + z_R(\omega^2 - \Omega^2) = q \qquad (20)$$

Put $z = z_R = fe^{(-ivt)}$

It is taken from reference [33] to solve the above equation

Differentiate 'z' with respective time

$dz / dt = [df / dt] e^{(-ivt)} + (-iv)fe^{(-ivt)}$ (A)

Once again differentiate 'dz / dt' with respective time

$d^2z / dt^2 = [d^2f / dt^2] e^{(-ivt)} + [df / dt] (-iv) e^{(-ivt)} + (-iv)\{[df / dt]e^{(-ivt)} + (-iv)fe^{(-ivt)}\}$

$= [d^2f / dt^2]e^{(-ivt)} - 2iv [df / dt] e^{(-ivt)} - v^2fe^{(-ivt)}$ (B)

Substituting (A) and (B) in (20)

$[d^2f / dt^2]e^{(-ivt)} - 2iv\, e^{(-ivt)} [df / dt] - v^2fe^{(-ivt)} + 2iv \{[df / dt] e^{(-ivt)} -ive^{(-ivt)} f\}$

$+ e^{(-ivt)}f (\omega^2 - v^2) = q$

$[d^2f / dt^2]e^{(-ivt)} + e^{(-ivt)} f\omega^2 = q$

$[d^2f / dt^{2]} + f\omega^2 = q\, e^{(ivt)}$

Above equation can be written in auxiliary equation form

$D^2 + \omega^2 = 0$

We can written as

$D = \pm i\omega$

Complete Solution = Complementary Function + Particular Integral

Complementary function (C.F) = $C_1 \cos (\omega t) + C_2 \sin (\omega t)$

Particular Integral (P.I) = $[q\, e^{(ivt)}] / (\omega^2 - v^2)$

$f = C_1 \cos (\omega t) + C_2 \sin (\omega t) + [q\, e^{(ivt)}] / (\omega^2 - v^2)$ --------------------------------(C)

By applying boundary conditions in (C)

f (0)=0 , Df (0.5) = 0

It can be written the equation as

$f = q / (\omega^2 - v^2)\{- \cos (\omega t) - [0.01917 + (iv)e^{(0.5iv)}] \sin (\omega t) +e^{(ivt)}$

$z = [qe^{(-ivt)}] / (\omega^2 - v^2) \{- \cos (\omega t) - [0.01917 + (iv)e^{(0.5iv)}] \sin (\omega t) +e^{(ivt)}\}$

The above equation is solved from the Linear Differential Method [34]. It is used to calculate the deflection of the rotor-shaft system.

Roots of A.E	Complementary function (C.F)
1) $m_1, m_2, m_3, \ldots$ (real and different roots)	$c_1 e^{m1x} + c_2 e^{m2x} + c_3 e^{m3x} + \ldots\ldots\ldots$
2) $m_1, m_1, m_3, \ldots$ (two real and equal roots)	$(c_1 + c_2 x)e^{m1x} + c_3 e^{m3x} + \ldots\ldots\ldots$
$m_1, m_1, m_4, \ldots$ (three real and equal roots)	$(c_1+c_2 x+ c_3x^2)e^{m1x} + c_4 e^{m4x} + \ldots\ldots\ldots$
4) $\alpha+i\beta, \alpha-i\beta, m_3, \ldots$ (a pair of imaginary roots)	$e^{ax}(c_1\cos\beta x+c_2\sin\beta x) +c_3 e^{m3x} + \ldots\ldots\ldots$
5) $\alpha\pm i\beta, \alpha\pm i\beta, m_4, \ldots$ (two pair of equal imaginary roots)	$e^{ax} [(c_1+ c_2 x) \cos \beta x + (c_3+ c_4 x) \sin \beta x)]+ c_5 e^{m5x} + \ldots\ldots\ldots$

SOLUTION TECHNIQUES ADAPTED

3.1. `LINEAR DIFFERENTIAL EQUATION METHOD:

Linear differential equations are those in which the dependent variable and its derivation occur only in the first degree and are not multiplied together. Thus the general linear differential equation of the n^{th} order of the form [34],

$$d^ny /dx^n + p_1 d^{n-1} y /dx^{n-1} + p_2 d^{n-2} y /dx^{n-2} + \ldots\ldots\ldots\ldots\ldots + p_n y=X ,$$

Where p_1, $p2, \ldots\ldots\ldots\ldots,$ p_n and X are functions of x only.
Linear differential equations with constant co-efficients are of the form

$$d^ny /dx^n + k_1 d^{n-1} y /dx^{n-1} + k_2 d^{n-2} y /dx^{n-2} +. \ldots\ldots\ldots\ldots\ldots+ k_n y=X ,$$

Where k_1, $k2, \ldots\ldots\ldots\ldots\ldots,$ kn are constants. Such equations are most important in the study electro-mechanical vibrations and other engineering problems.

Working procedure to solve equation:

$$d^ny /dx^n + k_1 d^{n-1} y /dx^{n-1} + k_2 d^{n-2} y /dx^{n-2} +. \ldots\ldots\ldots\ldots\ldots+ k_n y = X$$

of which the symbolic form is

$$(Dn + k_1 D^{n-1}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ kn_{-1} D+ k_n)y = X$$

Step 1:
To find the complementary function
(i)Write the auxiliary equation (A.E)

i.e. $D^n + k_1 D^{n-1}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+ k_{n-1} D+ k_n =0$, and solve it for D

(ii) Write the complementary function (C.F) as follows

Step 2:To find the particular integral
From symbolic form

$$P.I.= (1 / D^n + k_1 D^{n-1}+\ldots+ kn_{-1} D+ k_{n)} {}_X = [1/ f(D)] X \text{ or } [1/ Ø(D^2)]X.$$

(i)When $X = e^{ax}$

$P.I = [1/ f (D)] e^{ax,}$, put D = a

$= [1/ f (a)] e^{ax}$, provided $f(a) \neq 0$.

If $f(a) = 0$, the above rule fails and we proceed further.

$P.I = [1/ f(D)] e^{ax}$, put D = a

$= x [1/ f'(a)] e^{ax}$, provided $f'(a) \neq 0$.

If $f'(a) = 0$, the above rule fails and we proceed further.

$P.I = [1/ f(D)] e^{ax}$, put D = a

$= x^2 [1/ f''(a)] e^{ax}$, provided $f''(a) \neq 0$.

and so on

(ii) When X = sin(ax+b) or cos(ax+b).

$P.I = [1 / f(D^2)]\sin(ax+b)$ [or $\cos(ax+b)$], put $D^2 = -a^2$,providedf$(-a^2)\neq 0$,

$=[1 / f(-a^2)]\sin(ax+b)$ [or $\cos(ax+b)$], If $f(-a^2) = 0$, the above rule fails and we proceed further

P.I = $[1 / f'(D^2)] \sin(ax+b)$ [or $\cos(ax+b)$], put $D^2 = -a^2$, provided $f'(-a^2) \neq 0$

$= x [1 / f'(-a^2)] \sin(ax+b)$ [or $\cos(ax+b)$],

and so on

(iii)When $X = x^m$ **,** m being a positive integer

P.I = $[1/ f(D)]\, x^m = [f(D)]^{-1} x^m$,

Expand $[f(D)]^{-1}$ in ascending powers of D as far the term in D^m and operate on x^m term be term. Since the $(m+1)^{Th}$ and higher derivatives of x^m are zero, we need not consider terms beyond Dm**.**

(iv)When $X= e^{ax} V$, where V is a function of x.

P.I = $[1/ f(D)]\, e^{ax} V$

$= e^{ax} [1/ f(D+a)]V$

and the evaluate $[1/ f(D+a)]V$ as in (i),(ii),(iii).

(v)When X is any function of x

P.I =$[1/ f(D)]X$

Resolve $[1/ f(D)]$ into partial fractions and operate each partial fraction on X remembering that

$[1/ (D-a)]X = e^{ax} \int X e^{-ax}\, dx$

This method is a general one and can, therefore, be employed to obtain a particular integral any given case.

Step 3:

To find the complete solution (C.S)

Then the C.S, is y = C.F. + P.I.

3.2.Introduction to Finite Element Analysis

The finite element method [35] has become a powerful tool for the numerical solution of a wide range of engineering problems. Applications range from deformation and stress analysis of automotive, aircraft, building, and bridge structures to field analysis of heat flux, fluid flow, magnetic flux, seepage and other flow problems can be modeled with relative ease.

In this method of analysis, a complex region defining a continuum is discretized into simple geometric shapes called finite elements. The material properties and the governing relationships are considered over these elements and expressed in terms of unknown values at element corners. An assembly process, duly considering the loading and constraints, results in a set of equations. Solution of these equations gives us the approximate behavior of the continuum.

The finite element method is method of piecewise approximation in which the actual body of matter is considered as an integrated part of small elements known as finite elements. These elements are connected with one another at he joints called nodes or nodal points. Since the actual variation of the field variable like displacement, stress, temperature, pressure or velocity inside the continuum is not known the variation of the field variable inside the finite element can be approximated by a simple function called interpolation model which is defined in terms of field variables at nodes. Field equations can be written in the form of matrix equations and are solved for the nodal values of the field variables. The approximating functions define the variable through out the assemblage of finite elements.

The solution [36] of a general continuum problem by the finite element method always follows an orderly step-by-step process. With reference to static structural problems the step-by-step procedure can be stated as follows.

Discretization of the structure

The first step in the finite element method is to divide the structure or solution region into elements. Hence the structure is to be modeled with suitable finite elements. The number, size and arrangement of the elements are the input parameters.

Selection of proper interpolation or displacement model

Since the displacement solution of a complex structure under any specified load conditions cannot be predicted exactly, some suitable solution with in an element is assumed to approximate the unknown solution. The assumed solution must be simple from computational point of view, but should satisfy certain convergence requirements. In general the solution or the interpolation model is taken in the form of a polynomial.

Derivation of element stiffness matrix and load vectors

The stiffness matrix $[k^{(e)}]$ and the load vector $\{f^{(e)}\}$ of element 'e' are derived from the assumed displacement model by using either equilibrium conditions or a suitable variation principle.

Assemblage of element equations to detain the overall equilibrium equation

The individual element stiffness matrices and load vectors are assembled in a suitable manner, as the structure is an assemblage of these elements. The overall equilibrium equations are formulated as

$$[k]\{u\} = \{f\}$$

Where,

[k] is called the assembled stiffness matrix,

{u} is the vector of nodal displacements and

{f} is the vector of nodal forces for the complete structure.

Solution for the unknown nodal displacements

Structural analysis can be broadly divided as Static Analysis, Modal Analysis, Harmonic Analysis, Transient Dynamic Analysis, and Buckling Analysis.

(i) Static Analysis

Static analysis calculates the effects of steady loading conditions on a structure while ignoring inertia and damping effects, such as those caused by time-varying loads. Static analysis is used to determine the displacements, stresses, strains and forces in structures or components caused by loads that do not induce significant inertia and damping effects. Steady loading and response conditions are assumed; that is, the loads and structures response are assumed to vary slowly with respect to time.

The kinds of loading that can be applied in a static analysis include;

Externally applied forces and pressures

Steady-state inertial forces (such as gravity or rotational velocity)

Imposed (non-zero) displacements

Temperatures (for thermal strain)

Fluencies (for nuclear swelling)

The static analysis solution method is valid for all degrees of freedom (DOF‘s).

Inertial and damping effects are ignored, except for static acceleration fields.

(ii) Modal analysis:

Modal analysis is used to determine the vibration characteristics (natural frequencies and mode shapes) of a structure or a machine component while it is being designed. It also can be a starting point for another, more detailed, dynamic analysis, such as a transient dynamic analysis, a harmonic response analysis, or a spectrum analysis.

Modal analysis is to determine the natural frequencies and mode shapes of a structure. The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions.

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Performing a typical FEA analysis

The FEA program has many finite element analysis capabilities ranging from a simple, linear, static analysis to a complex, non-linear, transient dynamic analysis. A typical FEA analysis has three distinct steps

Build the model

Apply loads and obtain the solution

Review the results

Building a model

Modal Analysis of shaft-rotor system:

Modal analysis is to determine the vibration characteristics (natural frequencies and mode shapes) of a structure or a machine component while it is being designed. It also can be a starting point for another, more detailed, dynamic analysis, such as a transient dynamic analysis, a harmonic response analysis, or a spectrum analysis

Boundary Conditions

III. ARTIFICIAL NEURAL NETWORKS:

McCulloch and Pitts (1943) [37] developed the first artificial neuron. However, it was not until the psychologists David Rumelhart, of University of California at San Diego, and James McClelland, of Carnegie-Mellon University, developed the back propagation algorithm for training multi-layer perceptrons, that interest in ANNs, flourished (Rumelhart et al., 1986 a, b [39]; McClelland and Rumelhart, 1988[40]).

Recently, ANNs have been applied extensively to many prediction tasks. ANNs are able to determine the relationship between a set of input data and the corresponding output data without the need for predefined mathematical equations between these data. Neural networks have been applied in many applications such as: automotive, aerospace, banking, medical, robotics, electronic, and transportation.

Artificial neural networks perform well as hetero-associative classifiers and predictors, and are especially applicable when the data considered does not follow a known distribution or pattern. Past research in quality and process control have produced

neural networks which rival other, usually statistical, techniques in accuracy.
Moreover, the network approaches have superiority in robustness and convenience. Robustness comes about from a neural network's ability to handle noisy, corrupted or incomplete data. Convenience for process/quality control is gained from not having to pre-specify a technique or probability distribution, and being able to continuously adjust parameters. With the advent of hardwired neural networks, cost effective, real time predictive quality control will be available.

The structure and operation of natural neural networks (NNNs) have been described by many authors (Hertz et al., 1991; Zurada, 1992; Fausett, [43], 1994; Neuralware Inc., 1997). NNNs, of which the brain is an example, consist of billions of densely interconnected nerve cells called neurons. Each neuron receives the combined output signals of many other neurons through the synaptic gaps by input paths called dendrites. The dendrites collect the output signals and send them to the cell body, or the soma of the neuron, which sums the incoming signals. If the charge of the collected signals is strong enough, the neuron is activated and produces an output signal; otherwise the neuron remains inactive. The output signal is then transmitted to the neighbouring neurons through an output structure called the axon.

The axon of a neuron divides and connects to dendrites of the neighbouring neurons through junctions called synapses. Artificial neural networks (ANNs) are a form of artificial intelligence (AI), which in their architecture attempt to simulate the biological structure of the human brain and nervous system. Artificial neural networks (ANNs) are a form of artificial intelligence, which, in their architecture, try to simulate the biological structure of the human brain. ANNs try to mimic the behaviour of the basic biological and chemical processes of NNNs. ANNs learn "by example" and therefore are well suited to complex processes where the relationship between the variables is unknown. ANNs consist of a number of artificial neurons (variously known as "processing elements", "PEs", "Nodes" or "Units") representative of the neurons in ANNs. Each processing element has several input paths and one output path, as shown in Figure 2 An individual PE receives its inputs from many other processing elements via weighted input connections. These weighted inputs are summed and passed through a transfer function to produce a single activation level for the processing element,
which is the node output.

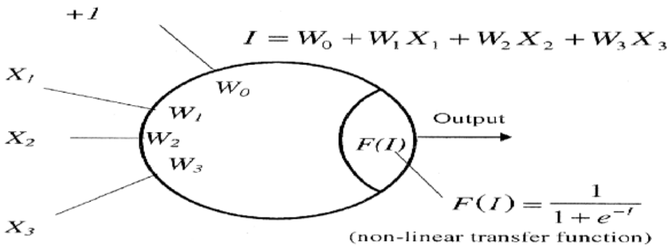


Figure 2. Typical processing element (PE) in a neuron.

A typical structure of artificial neural networks consists of many processing elements that are arranged in layers: an input layer, an output layer, and one or more layers inbetween, called intermediate or hidden layers (Figure 3). Each processing element in a specific layer is interconnected to all the processing elements in the next layer via weighted connections. The scalar weights determine the strength of the connection between interconnected neurons. A zero weight refers to no connection between two neurons and a negative weight refers to a prohibitive relationship.

The propagation of information starts at the input layer where the input data are
presented. The inputs are weighted and received by each node in the next layer. The weighted inputs are then summed and passed through a non-linear transfer function to produce the node output, which is weighted and passed to the processing elements in the next layer. The network's output is compared with the actual value and the error between the two values is calculated. This error is then used to adjust the weights until the network can find a set of weights that will produce the input-output mapping with the smallest possible error.

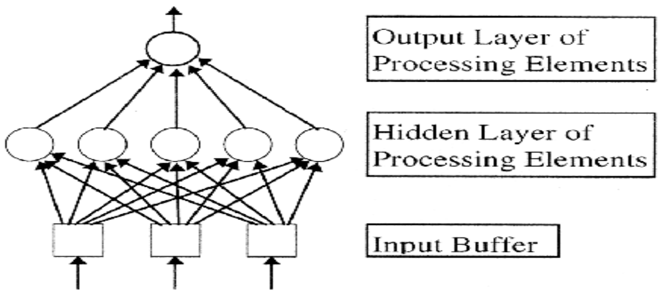


Figure 3. Typical structure of ANN

Back-propagation neural networks are adopted in this work, as they have a high capability of data mapping . Back-propagation neural networks have been applied to a wide range of areas including classification, estimation, prediction, and functions synthesis and they are currently the most widely used neural network. The topology and algorithm details of back-propagation neural networks are beyond the scope of this report and can be found in many publications. Back propagation neural networks are one of the most common neural network structures, as they are simple and effective, and have found home in a wide assortment of machine learning applications.

Supervised learning compares the obtained output with the expected target value and adapts the network to obtain a stable weight structure. In present work, a back propagation neural network is used for amplitudes estimation.

The data for training the networks are obtained from the literature reference [1]. . The obtained data are normalized; i.e. each entity of a particular input vector is divided by the length of the vector to scale it down to 0 to1 to prevent the abnormal growth of weight structures during successive iterations. These data are fed to the network. Weights are updated using the generalized Delta rule.

$W_{new} = W_{old} - \alpha E_T I.$

Where

W_{new}= weight after modification.

W_{old} = weight structure before modification.

α = learning rate, usually taken between 0 and 1.

E_T = error obtained.

Weight change is calculated for all connections. Errors for all patterns are summed and the algorithm is run until the error falls below a specified value.

To reach the global minimum of the error of the network, the aid of heuristic optimization techniques is sought in present work. The present work uses the concept of momentum to overcome local minima .The technique lies in adding a portion of previous weight changes in weight modification:

$W_2(k+1) = W_2(k) + \alpha E_T O_1 + \rho \Delta W_2(k)$

Where

ρ=momentum rate, usually taken to be around 0.5 - 0.8.

k = number of the present iterations.

The obtained stable weight structures are used with new input patterns to obtain the amplitude values.

IV.RESULTS AND DISCUSSION:

Classical Method (LDM) Results:

$z = [qe^{(-ivt)}] / (\omega^2 - v^2) \{- \cos(\omega t) - [0.01917 + (iv)e^{(0.5iv)}] \sin(\omega t) + e^{(ivt)}\}$

where z = x + iy

by varying the speed of the shaft from 50 rpm to 1800 rpm deflection of the shaft can be calculated from the above equation. Dimensions of the rotor-shaft system:

Diameter of shaft	: 0.0145 m
Length of the shaft	: 1 m
Rotor disc weight	: 1.64 kg
Young's modulus of steel	: 1.9×10^{11} N / m^2

S.No.	Speed (rpm)	Deflection (mm)
1	50	0.04402489
2	100	0.214086205
3	150	0.000426717
4	200	0.004804517
5	250	0.005774132
6	300	0.154791801
7	350	0.095430033
8	400	0.09432363
9	450	0.124408639
10	500	0.211394707
11	550	0.005741812

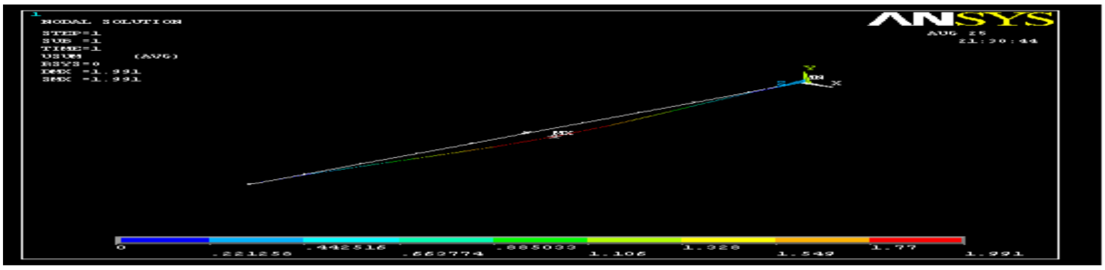
12	600	0.168263215
13	650	0.27036401
14	700	0.286582197
15	750	0.092722227
16	800	0.306905844
17	850	0.201956467
18	900	0.148668497
19	950	0.197481519
20	1000	1.786671022
21	1050	1.962335019
22	1100	1.511730089
23	1150	0.702974932
24	1200	0.212774319
25	1250	0.321679591
26	1300	0.126031088
27	1350	0.022767366
28	1400	0.168263215
29	1450	0.27036401
30	1500	0.286582197
31	1550	0.092722227
32	1600	0.306905844
33	1650	0.201956467
34	1700	0.148668497
35	1750	0.197481519
36	1800	0.154791801

Static Analysis Results: In FEA by giving input dimensions of rotating shaft system, varying the speed of the shaft from 50 rpm to 1800 rpm deflection of the shaft can be obtained and compared.
Dimensions of the rotor-shaft system:

Diameter of shaft : 0.0145 m
Length of the shaft : 1 m
Rotor disc weight : 1.64 kg
Young's modulus of steel : 1.9×10^{11} N / m^2

S.No.	Steady inertia load (Angular velocity) (rad/sec)	FEA deflection values (mm)
1	5.236	0.03667
2	10.472	0.24003
3	15.707	0.00039
4	20.944	0.00592
5	26.18	0.00519
6	31.416	0.17677
7	36.652	0.10898
8	41.888	0.11602
9	47.124	0.10898
10	52.359	0.23934

11	57.596	0.00508
12	62.832	0.15442
13	68.067	0.23657
14	73.303	0.3531
15	78.539	0.08148
16	83.775	0.34718
17	89.011	0.19085
18	94.247	0.16829
19	99.483	0.1709
20	104.719	1.65898
21	109.955	1.99087
22	115.191	1.54366
23	120.427	0.65266
24	125.663	0.2579
25	130.899	0.30051
26	136.135	0.15276
27	141.371	0.02102
28	146.607	0.15085
29	151.843	0.23757
30	157.079	0.35
31	162.315	0.08968
32	167.551	0.35767
33	172.787	0.19553
34	178.023	0.17513
35	183.259	0.19096
36	188.495	0.14136

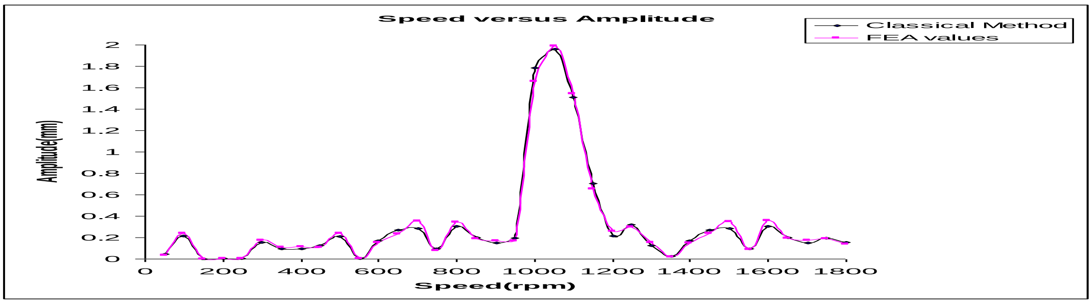


Above one is FEA output with speed 1050 rpm,deflection value is 1.991 mm is obtained

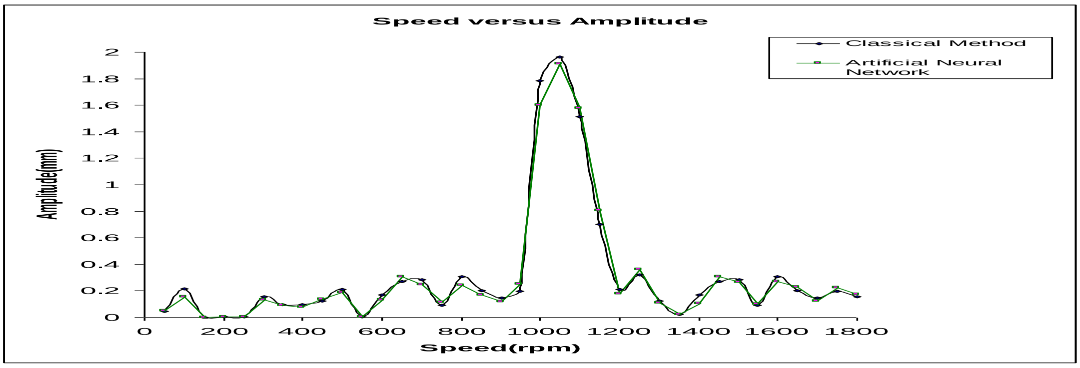
Artificial Neural Network Results: In ANN a back propagation algorithm is used to analyze deflection by giving input values as speed to the network ,deflection values are obtained.

S.No.	Speed(rpm)	ANN deflection values
1	50	0.050364474
2	100	0.154955595
3	150	0.000488164
4	200	0.00437211
5	250	0.006680671
6	300	0.131108655
7	350	0.091803692
8	400	0.078307478
9	450	0.135854234
10	500	0.186598108

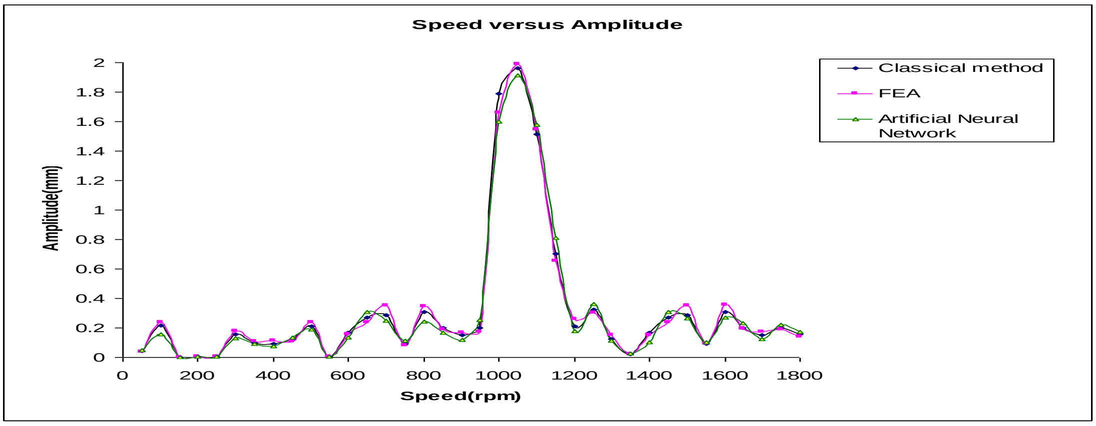
11	550	0.006568633
12	600	0.133096203
13	650	0.305781695
14	700	0.247629945
15	750	0.113668178
16	800	0.241086817
17	850	0.16802778
18	900	0.117596781
19	950	0.251591455
20	1000	1.600857236
21	1050	1.913276643
22	1100	1.576734483
23	1150	0.808315726
24	1200	0.177241008
25	1250	0.363819618
26	1300	0.111159419
27	1350	0.024861963
28	1400	0.104827983
29	1450	0.309296427
30	1500	0.265243287
31	1550	0.100047283
32	1600	0.268542613
33	1650	0.231038198
34	1700	0.121759499
35	1750	0.223351598
36	1800	0.173366817



Comparison graph is drawn between speed versus amplitude for LDM & FEA values it is observed that at speed 1050 rpm, maximum deflection value is observed.



Comparison graph is drawn between speed versus amplitude for LDM & Artificial Neural Networks (ANN) values it is observed that at speed 1050 rpm, maximum deflection value is observed.



Above graph is comparison graph between three methods LDM, FEA, ANN

The dimensions of the rotor-shaft system are taken from the Reference [1].

Dimensions of the rotor-shaft system:

Diameter of shaft	: 0.0145 m
Length of the shaft	: 1 m
Rotor disc weight	: 1.64 kg
Young's modulus of steel	: $1.9\text{x}10^{11}$ N / m^2

Calculating stiffness of the shaft:

$$k = [48EI] / L^3$$
$$= [48E (\pi d^4)] / (64\ L^3)$$
$$= [48\text{x}1.9\text{x}10^{11}\text{x}\ \pi\ \text{x}\ (0.0145)^4] / (64\ \text{x}\ (1^3))$$
$$= 19789.58769 \quad N / m$$

Using the above equation, the theoretical natural frequency can be calculated as

$$\omega_n = \sqrt{(k / m)}$$
$$= \sqrt{(19789.58769 / 1.64)}$$
$$\omega_n = 109.8490863 \text{ rad / sec}$$

f $= \omega_n / 2\pi$ Hz

$$= 109.8490863 / \mathbf{2}\,\pi$$

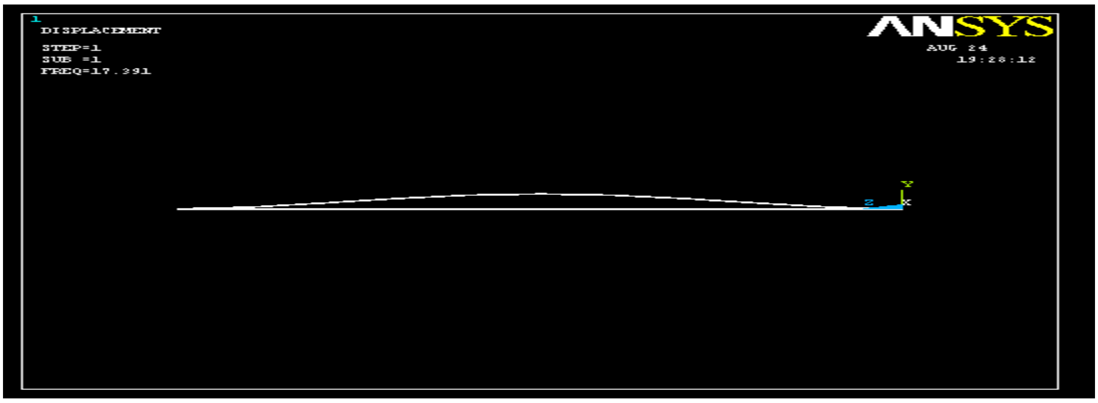
f= 17.48302508 Hz

Modal Analysis Results:

In FEA modal analysis of rotating machinery system is done, natural frequency values is obtained and compared with theoretical values.

Table: Natural frequency values

S.No	Natural frequencies (Hz)
1	17.391
2	50.284
3	107.63
4	197.95
5	348.20
6	591.77



Above one is FEA output first natural frequency is 17.391 Hz is obtained

V. CONCLUSIONS AND FUTURE SCOPE OF WORK

This projects mainly deals with the analysis of rotor-shaft system as explained previous. That is Classical Method, Static analysis, ANN and Modal analysis. The results obtained are concluded as below

- The calculated amplitude from Classical method of the rotor-shaft system is high at the critical speed and low at other running speed.
- The results obtained in Static analysis matches to the results obtained of the Classical method (LDM)
- The frequency results obtained from Modal analysis are nearer to the theoretical results.
- The results obtained from ANN are nearer to the Classical method. Hence the network systems could be used as an analyzer for rotating-machinery systems.

VI. FUTURE SCOPE OF WORK:

The mathematical model of the rotor-shaft system can be extended to

- The simple supported beam with damping for single mass on midspan
- The simple supported beam with/without damping for different masses on midspan.

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